GACE® Mathematics Assessment Test I (022)
Curriculum Crosswalk

	Required Coursework Numbers														
Subarea I. Number and Quantity (30%)															
Objective 1: Understands and applies knowledge of the real number system and vector and matrix quantities															
A. Understands the properties of exponents															
• Performs operations involving exponents, including negative and rational exponents															
• Demonstrates an understanding of the properties of exponential expressions															
• Uses the properties of exponents to rewrite expressions that have radicals or rational exponents															
B. Understands the properties of rational and irrational numbers and the interactions between those sets of numbers															
• Recognizes that the sum or product of two rational numbers is rational															
• Recognizes that the sum of a rational number and an irrational number is irrational															
• Recognizes that the product of a nonzero rational number and an irrational number is irrational															
• Recognizes that the sum or product of two irrational numbers can be rational or irrational															

	Required Coursework Numbers														
C. Is familiar with the representation and modeling of vector quantities and how operations on vectors are performed															
• Represents vector quantities by directed line segments and uses appropriate symbols for vectors and their magnitudes															
• Finds the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point															
• Solves problems involving velocity and other quantities that can be represented by vectors															
• Adds vectors end-to-end, component-wise, and by the parallelogram rule															
• Given two vectors in magnitude and direction form, determines the magnitude and direction of their sum															
D. Understands how to perform operations on matrices and how to use matrices in applications															
• Uses matrices to represent and manipulate data															
• Multiplies matrices by scalars to produce new matrices															
• Adds, subtracts, and multiplies matrices of appropriate dimensions															
• Understands that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties															

	Required Coursework Numbers														
• Understands the role played by zero and identity matrices in matrix addition and multiplication															
• Understands that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse															
E. Understands how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions															
• Applies the concept of a ratio and uses ratio language and notation to describe a relationship between two quantities and solve problems															
• Uses ratio reasoning to convert rates															
• Solves problems involving scale factors															
• Recognizes and represents proportional and inversely proportional relationships between two quantities															
• Uses proportional relationships to solve multistep ratio, average, and percent problems															
• Solves measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate															
• Converts units within the metric and customary systems															

	Required Coursework Numbers														
F. Understands various ways to represent, compare, estimate, and perform calculations on very large and very small numbers; e.g., scientific notation, orders of magnitude															
• Represents and compares very large and very small numbers															
• Uses orders of magnitude to estimate very large and very small numbers															
• Performs calculations on numbers in scientific notation															
Objective 2: Understands and applies knowledge of quantities and the complex number system															
A. Understands how to solve problems by reasoning quantitatively; e.g., dimensional analysis, reasonableness of solutions															
• Uses units as a way to understand problems and to guide the solution of multistep problems															
• Chooses and interprets units consistently in formulas															
• Chooses and interprets the scale and the origin in graphs and data displays															
• Recognizes the reasonableness of results within the context of a given problem															
B. Understands the structure of the natural, integer, rational, real, and complex number systems and how the basic operations (+, −, ×, and ÷) on numbers in these systems are performed															
• Solves problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers															

	Required Coursework Numbers														
• Applies the order of operations															
• Given operations on a number system, determines whether the properties (e.g., commutative, associative, distributive) hold															
• Compares, classifies, and orders real numbers															
• Demonstrates an understanding of the properties of counting numbers; e.g., prime, composite, prime factorization, even, odd, factors, multiples															
C. Knows how complex numbers and operations on them are represented in the complex plane															
• Represents complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers)															
• Explains why the rectangular and polar forms of a given complex number represent the same number															
• Represents addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane, and uses properties of the representation for computation															
• Calculates the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints															

	Required Coursework Numbers														
D. Understands how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions															
• Solves quadratic equations with real coefficients that have complex solutions															
• Extends polynomial identities to the complex numbers; e.g., $x^2 + y^2 = (x + yi)(x - yi)$															
E. Knows how to analyze both precision and accuracy in measurement situations															
• Chooses a level of accuracy appropriate to limitations on measurement when reporting quantities															
• Calculates or estimates absolute and relative error in the numerical answer to a problem															
Subarea II. Algebra (40%)															
Objective 1: Sees structure in expressions and understands arithmetic with polynomials and rational expressions															
A. Understands how to write algebraic expressions in equivalent forms															
• Uses the structure of an expression to identify ways to rewrite it															
• Understands how to rewrite quadratic expressions for specific purposes; e.g., factoring/finding zeros, completing the square/finding maxima or minima															

	Required Coursework Numbers														
• Uses the properties of exponents to rewrite expressions for exponential functions															
B. Understands how to perform arithmetic operations on polynomials															
• Adds, subtracts, multiplies, and divides polynomials															
C. Understands the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions															
• Knows and applies the remainder theorem: for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$															
• Uses factorization to identify zeros of polynomials															
• Uses zeros of a polynomial to construct a rough graph of the function defined by the polynomial															
D. Understands how to use the binomial theorem to solve problems															
• Applies the binomial theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n															
E. Understands how to rewrite rational expressions and perform arithmetic operations on rational expressions															
• Rewrites simple rational expressions in different forms															

	Required Coursework Numbers														
• Understands that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression															
• Adds, subtracts, multiplies, and divides rational expressions															
F. Understands the properties of number systems under various operations															
• Given operations on algebraic expressions, determines whether properties (e.g., commutative, associative, distributive) hold															
• Performs calculations using newly defined functions															
Objective 2: Understands how to create equations and how to reason with equations and inequalities															
A. Understands how to create equations and inequalities that describe relationships															
• Creates equations and inequalities in one variable and uses them to solve problems and graph solutions on the number line															
• Creates equations and inequalities to represent relationships between quantities, solves problems, and graphs them on the coordinate plane with labels and scales															
• Represents constraints by equations, inequalities, or systems of equations and/or inequalities, and interprets solutions as viable or nonviable options in a modeling context															

	Required Coursework Numbers														
• Rearranges formulas to highlight a quantity of interest; e.g., solve $d = rt$ for t															
B. Understands how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions															
• States each step in solving a simple equation															
• Solves simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions															
C. Understands how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities															
• Solves linear equations and inequalities, including equations with coefficients represented by letters															
• Uses the method of completing the square to transform any quadratic equation in x into the equivalent form $(x - p)^2 = q$															
• Solves equations using a variety of methods (e.g., using graphs, using the quadratic formula, factoring)															
• Uses different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation															
D. Understands how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities															

	Required Coursework Numbers														
• Explains why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system															
• Solves a system consisting of two linear equations in two variables algebraically and graphically															
• Solves a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically															
• Represents a system of linear equations as a single matrix equation															
• Finds the inverse of a matrix if it exists and uses it to solve systems of linear equations															
• Explains why the x-coordinates of the intersection points of the graphs of $y = f(x)$ and $y = g(x)$ are the solutions of $f(x) = g(x)$															
• Finds the solutions of $f(x) = g(x)$ approximately (e.g., uses technology to graph the functions, makes tables of values, finds successive approximations); includes cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, or logarithmic functions															
• Graphs the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graphs the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes															

	Required Coursework Numbers														
E. Understands the concept of rate of change of nonlinear functions															
• Calculates and interprets the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval															
F. Understands the concepts of intercept(s) of a line and slope as a rate of change															
• Calculates and interprets the intercepts of a line															
• Calculates and interprets the slope of a line presented symbolically, numerically, or graphically															
• Estimates the rate of change of a linear function from a graph															
G. Understands how to find the zero(s) of functions															
• Uses a variety of techniques to find and analyze the zero(s) (real and complex) of functions															
Subarea III. Discrete Mathematics and Calculus (30%)															
Objective 1: Understands and applies knowledge of discrete mathematics															
A. Understands sequences; e.g., arithmetic, recursively defined, geometric															
• Writes arithmetic and geometric sequences both recursively and with an explicit formula, uses them to model situations, and translates between the two forms															

	Required Coursework Numbers														
• Evaluates, extends, or algebraically represents rules that involve number patterns															
• Explores patterns in order to make conjectures, predictions, or generalizations															
B. Understands the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena															
• Understands the differences between discrete and continuous representations; e.g., data, functions															
• Understands how discrete and continuous representations can be used to model various phenomena															
C. Knows how to model and solve problems using vertex-edge graphs, trees, and networks															
• Constructs, uses, and interprets simple diagrams to solve problems															
• Solves linear programming problems															
D. Understands basic terminology and symbols of logic															
• Understands the basic terminology of logic															
• Uses logic to evaluate the truth of statements															
• Uses logic to evaluate the equivalence of statements; e.g., statement and contrapositive															

	Required Coursework Numbers														
• Identifies basic properties of quantifiers; e.g., for all, there exists															
• Negates statements involving quantifiers; e.g., for all, there exists															
E. Understands how to use counting techniques such as the multiplication principle, permutations, and combinations															
• Uses counting techniques to solve problems															
F. Understands basic set theory; e.g., unions, differences, and Venn diagrams															
• Solves problems using basic set theory; i.e., union, intersection, complement, difference															
• Uses Venn diagrams to answer questions about sets															
Objective 2: Understands calculus concepts and applies knowledge to solve calculus problems															
A. Understands the meaning of a limit of a function and how to calculate limits of functions, how to determine when the limit does not exist, and how to solve problems using the properties of limits															
• Graphically analyzes the limit of $f(x)$ as x approaches a fixed value from both left and right															
• Solves limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that x is approaching															

	Required Coursework Numbers														
• Analyzes one-sided limits for various functions to see whether or not the limit exists															
• Recognizes limits that do not exist, such as $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ and $\lim_{x\to 0} \frac{1}{\sqrt[3]{x^2}}$															
B. Understands the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change															
• Constructs a function graph for a given function and a given point (a, $f(a)$), and explains what happens to the succession of slopes of secant lines connecting (a, $f(a)$) to (x, $f(x)$) as x approaches a, from both the right side and the left side															
• Uses the limit definition of the derivative to find the derivative of a given function at a given value of x and to find the derivative function															
C. Understands how to show that a particular function is continuous															
• Applies the three steps (i.e, $f(a)$ exists, $\lim_{x\to a} f(x)$ exists, and $f(a) = \lim_{x\to a} f(x)$) that are part of the definition of what it means for a function to be continuous at $x = a$ to verify whether a given function is continuous at a given point															
D. Knows the relationship between continuity and differentiability															
• Gives examples of functions that are continuous at $x = a$ but not differentiable at $x = a$, and explains why															

	Required Coursework Numbers														
E. Understands how and when to use standard differentiation and integration techniques															
• Uses standard differentiation techniques															
• Uses standard integration techniques															
• Understands the relationship between position, velocity, and acceleration functions of a particle in motion															
F. Understands how to analyze the behavior of a function; e.g., extrema, concavity, symmetry															
• Uses the first and second derivatives to analyze the graph of a function															
G. Understands how to apply derivatives to solve problems; e.g., related rates, optimization															
• Applies derivatives to solve problems															
H. Understands the foundational theorems of calculus; e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem															
• Solves problems using the foundational theorems of calculus															
• Understands the relationship between differentiation and integration, including the role of the fundamental theorems of calculus															
• Matches graphs of functions with graphs of their derivatives or accumulations															
• Understands how to use differentiation and integration of a function to express rates of change and total change															

	Required Coursework Numbers														
• Understands and calculates the average value of a function over an interval; i.e., mean value theorem of integrals															
I. Understands how to use integration to compute area, volume, distance, or other accumulation processes															
• Uses integration techniques to compute area, volume, distance, or other accumulation processes															
J. Knows how to determine the limits of sequences, if they exist															
• Determines the limits of sequences when they exist															
K. Is familiar with simple infinite series															
• Determines if simple infinite series converge or diverge															
• Finds the sum of a simple infinite series if it exists															
• Finds the partial sum of a simple infinite series															
• Models phenomena (e.g., compound interest, annuities, growth, decay) using finite and infinite arithmetic and geometric sequences and series															