NOTATIONS

(a, b) \{ x : a < x < b \}
[a, b) \{ x : a \leq x < b \}
(a, b] \{ x : a < x \leq b \}
[a, b] \{ x : a \leq x \leq b \}
gcd(m, n) \text{ greatest common divisor of two integers } m \text{ and } n
lcm(m, n) \text{ least common multiple of two integers } m \text{ and } n
[x] \text{ greatest integer } m \text{ such that } m \leq x
m \equiv k \text{ (mod } n) \text{ } m \text{ and } k \text{ are congruent modulo } n \text{ ( } m \text{ and } k \text{ have the same remainder when divided by } n, \text{ or equivalently, } m - k \text{ is a multiple of } n)\]
\[ f^{-1} \text{ inverse of an invertible function } f \text{ (not the same as } \frac{1}{f}) \]
\[ \lim_{x \to a^+} f(x) \text{ right-hand limit of } f(x); \text{ limit of } f(x) \text{ as } x \text{ approaches } a \text{ from the right} \]
\[ \lim_{x \to a^-} f(x) \text{ left-hand limit of } f(x); \text{ limit of } f(x) \text{ as } x \text{ approaches } a \text{ from the left} \]
\[ \emptyset \text{ the empty set} \]
\[ x \in S \text{ } x \text{ is an element of set } S \]
\[ S \subset T \text{ } \text{set } S \text{ is a proper subset of set } T \]
\[ S \subseteq T \text{ either set } S \text{ is a proper subset of set } T \text{ or } S = T \]
\[ S \cup T \text{ union of sets } S \text{ and } T \]
\[ S \cap T \text{ intersection of sets } S \text{ and } T \]

DEFINITIONS

A relation $\mathcal{R}$ on a set $S$ is

reflexive if $x \mathcal{R} x$ for all $x \in S$
symmetric if $x \mathcal{R} y \Rightarrow y \mathcal{R} x$ for all $x, y \in S$
transitive if $(x \mathcal{R} y \text{ and } y \mathcal{R} z) \Rightarrow x \mathcal{R} z$ for all $x, y, z \in S$
antisymmetric if $(x \mathcal{R} y \text{ and } y \mathcal{R} x) \Rightarrow x = y$ for all $x, y \in S$

An equivalence relation is a reflexive, symmetric, and transitive relation.
FORMULAS

Sum
\[
\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \\
\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \\
\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}
\]

Half-angle \(\left(\text{sign depends on the quadrant of } \frac{\theta}{2}\right)\)
\[
\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \\
\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}
\]

Range of Inverse Trigonometric Functions
\[
\sin^{-1} x \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\
\cos^{-1} x \quad [0, \pi] \\
\tan^{-1} x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)
\]

Law of Sines
\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

Law of Cosines
\[
c^2 = a^2 + b^2 - 2ab \cos C
\]

DeMoivre’s Theorem
\[
(cos \theta + i \sin \theta)^k = \cos(k \theta) + i \sin(k \theta)
\]
Coordinate Transformation

Rectangular \((x, y)\) to polar \((r, \theta)\):  \(r^2 = x^2 + y^2; \quad \tan \theta = \frac{y}{x} \) if \(x \neq 0\)

Polar \((r, \theta)\) to rectangular \((x, y)\):  \(x = r \cos \theta; \quad y = r \sin \theta\)

Distance from point \((x_1, y_1)\) to line \(Ax + By + C = 0\)

\[
d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}
\]

Volume

Sphere with radius \(r\):  \(V = \frac{4}{3} \pi r^3\)

Right circular cone with height \(h\) and base of radius \(r\):  \(V = \frac{1}{3} \pi r^2 h\)

Right circular cylinder with height \(h\) and base of radius \(r\):  \(V = \pi r^2 h\)

Pyramid with height \(h\) and base of area \(B\):  \(V = \frac{1}{3} Bh\)

Right prism with height \(h\) and base of area \(B\):  \(V = Bh\)

Surface Area

Sphere with radius \(r\):  \(A = 4 \pi r^2\)

Right circular cone with radius \(r\) and slant height \(s\):  \(A = \pi rs + \pi r^2\)

Differentiation

\[
(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)
\]

\[
(f(g(x)))' = f'(g(x))g'(x)
\]

\[
\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \quad \text{if} \ g(x) \neq 0
\]

Integration by Parts

\[
\int u \, dv = uv - \int v \, du
\]